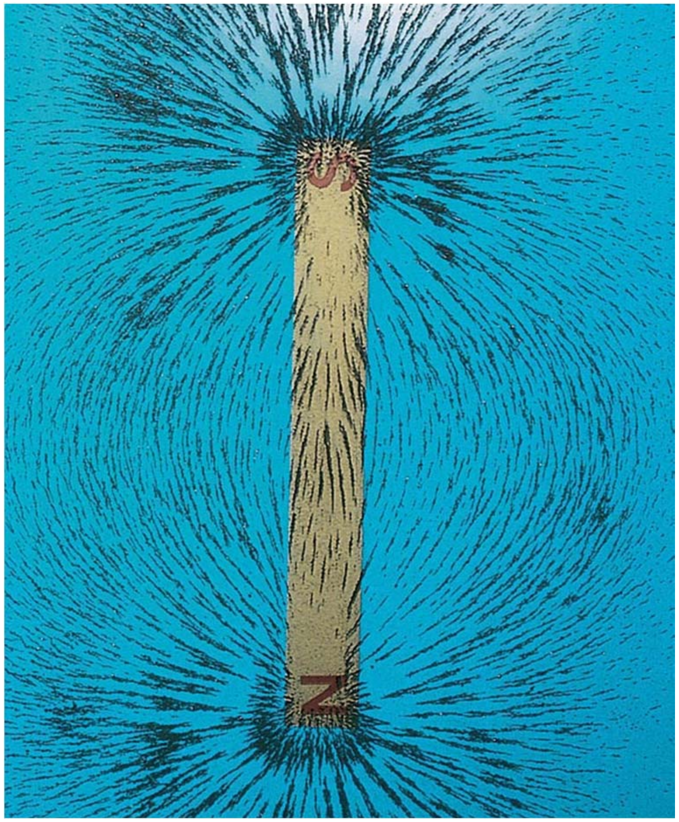
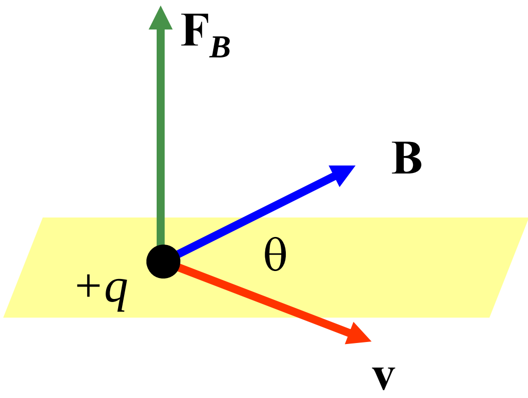
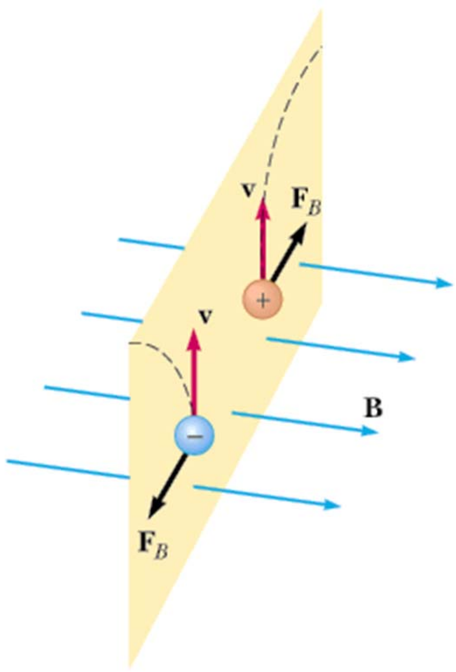
Magnetic Fields

- Certain iron containing materials have been known to attract or repel each other.
- Compasses align to the magnetic field of earth.
- Analogous to positive and negative charges, every magnet has a north and a south pole (but always as a pair).
- Magnetic fields can be created by electrical currents.



Magnetic Field and Magnetic Force





$$\left|\vec{\mathbf{F}}_B\right| \propto q, \left|\vec{\mathbf{v}}\right|$$

$$\vec{\mathbf{v}} \,//\, \vec{\mathbf{B}} \Rightarrow \vec{\mathbf{F}}_B = 0$$

$$\theta \neq 0 \Rightarrow \vec{\mathbf{F}}_B \perp \vec{\mathbf{B}}, \vec{\mathbf{v}}$$

$\vec{\mathbf{F}}_B\left(q < 0, \vec{\mathbf{v}}\right)$ opposite direction $\vec{\mathbf{F}}_B\left(q > 0, \vec{\mathbf{v}}\right)$

$$\left|\vec{\mathbf{F}}_B\right| \propto \sin\theta$$

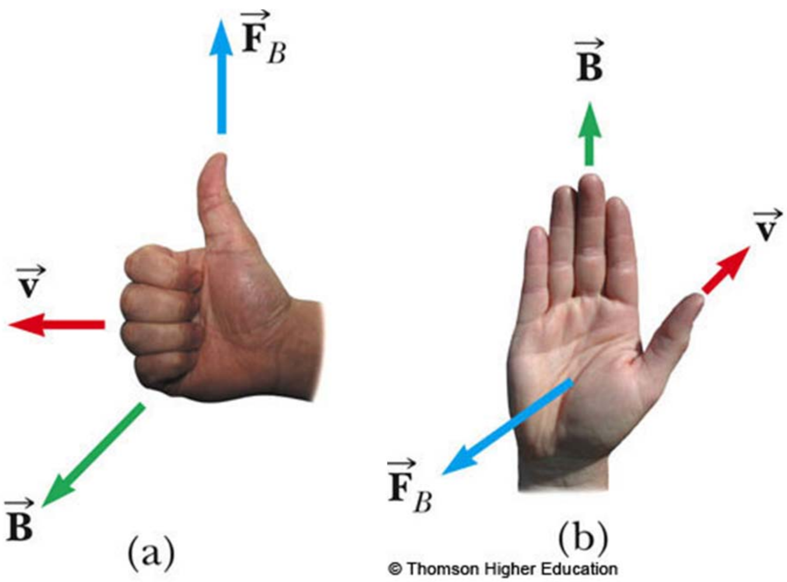
Magnetic Force

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

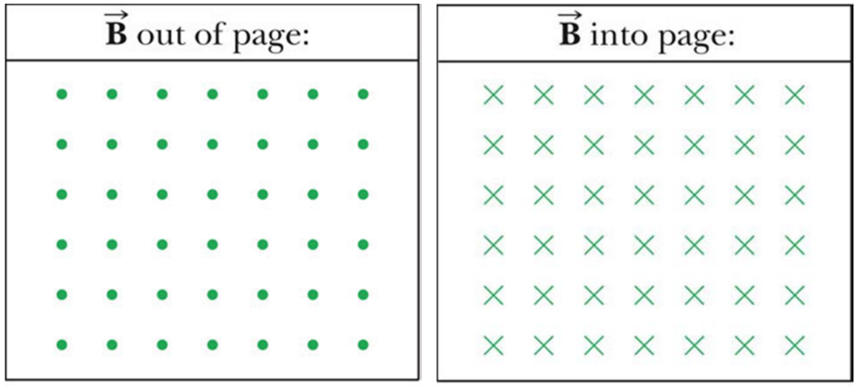
$$F_B = |q| vB \sin\theta$$

Magnetic Field, B: Tesla (T)=N/(C.m/s)=N/(A.m)

Right Hand Rule



Magnetic Field Representation



Electric and Magnetic Forces

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} \qquad \vec{\mathbf{F}}_E = q\vec{\mathbf{E}}$$

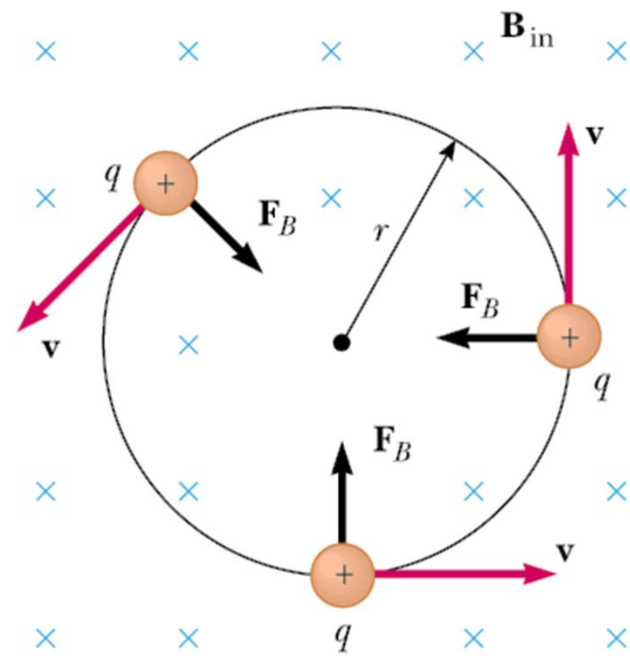
- Electric forces act in the direction of the electric field, magnetic forces are perpendicular to the magnetic field.
- A magnetic force exists only for charges in motion.
- The magnetic force of a steady magnetic field does no work when displacing a charged particle.
- The magnetic field can alter the direction of a moving charged particle but not its speed or its kinetic energy.

Motion of Charged Particles in a Uniform Magnetic Field

Consider a positive charge moving perpendicular to a magnetic field with an initial velocity, **v**.

The force F_B is always at right angles to v and its magnitude is,

$$F_B = qvB$$



So, as *q* moves, it will rotate about a circle and $\mathbf{F}_B$ and **v** will always be perpendicular. The magnitude of **v** will always be the same, only its direction will change.

Cyclotron Frequency

To find the radius and frequency of the rotation:

The radial force $\sum F = ma_r$

$$F_B = qvB = m\frac{v^2}{r}$$

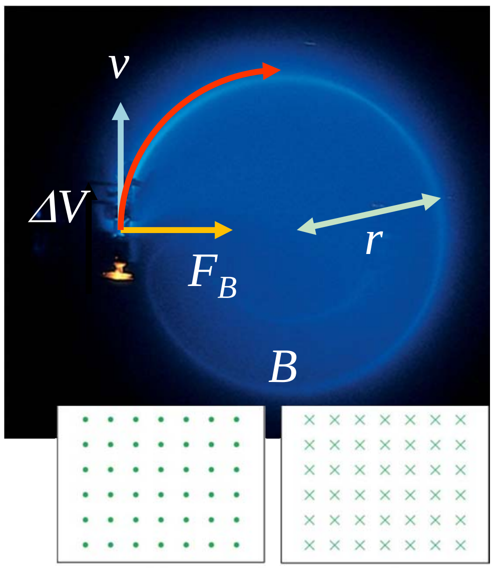
$$r = \frac{mv}{qB}$$

The angular speed $\omega = \frac{v}{r} = \frac{qB}{m}$ ⟶ Cyclotron frequency

The period $T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$

Example 29.3

Electron beam in a magnetic field



ΔV = 350 V

r = 7.5 cm

$q = e$ = 1.6 x 10^{-19} C

$m = m_e$ = 9.11 x 10^{-31} kg

B = ?

ω = ?

$$\Delta K = U$$

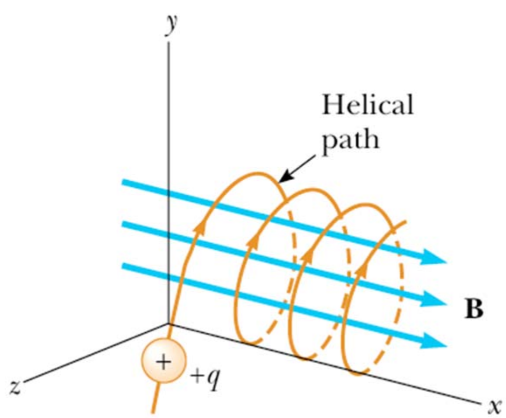
$$\frac{1}{2} m_e v^2 = e\Delta V$$

$$v = \sqrt{\frac{2e\Delta V}{m_e}} = 1.11 \times 10^7 \, m/s$$

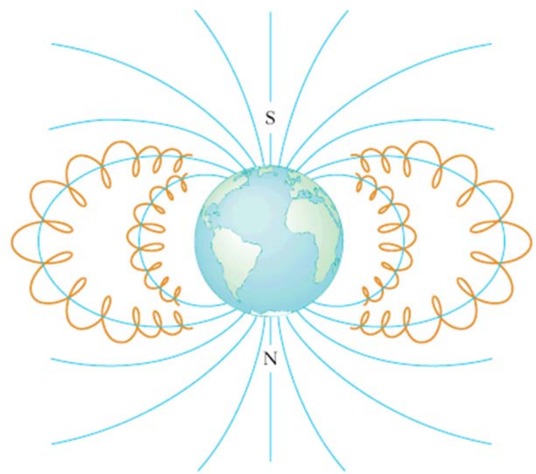
$$r = \frac{mv}{qB} \longrightarrow B = \frac{m_e v}{er} = 8.4 \times 10^{-4} T$$

$$\omega = \frac{qB}{m} = \frac{eB}{m_e} = 1.5 \times 10^8 \, rad/s$$

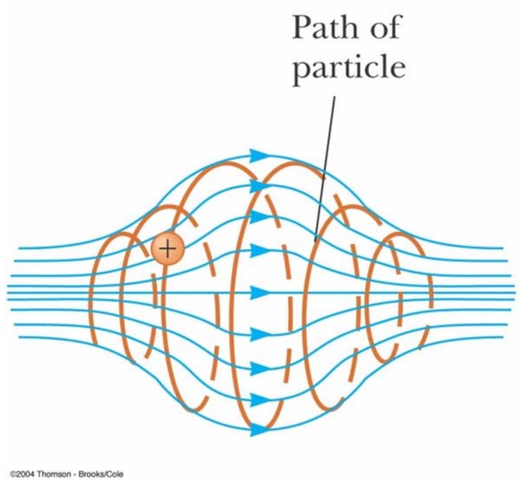
Guiding Charged Particles



Helical Motion –
v_x, v_y and v_z



Cosmic Rays in the
Van Allen Belt



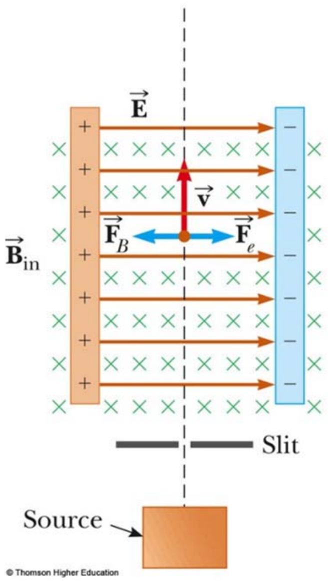
Complex Fields –
Magnetic Bottles

Lorenz Force and Applications

Essentially it is the total electric and magnetic force acting upon a charge.

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

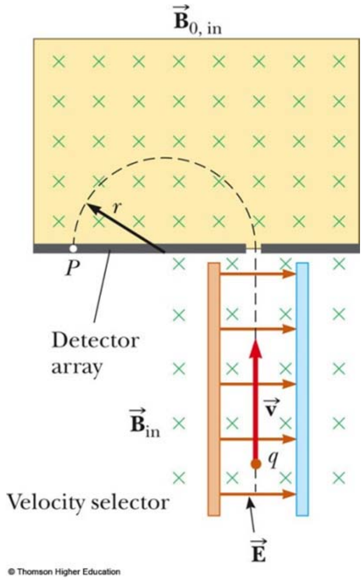
Velocity Selector



$$F_e = F_B$$

$$v = \frac{E}{B}$$

Mass Spectrometer

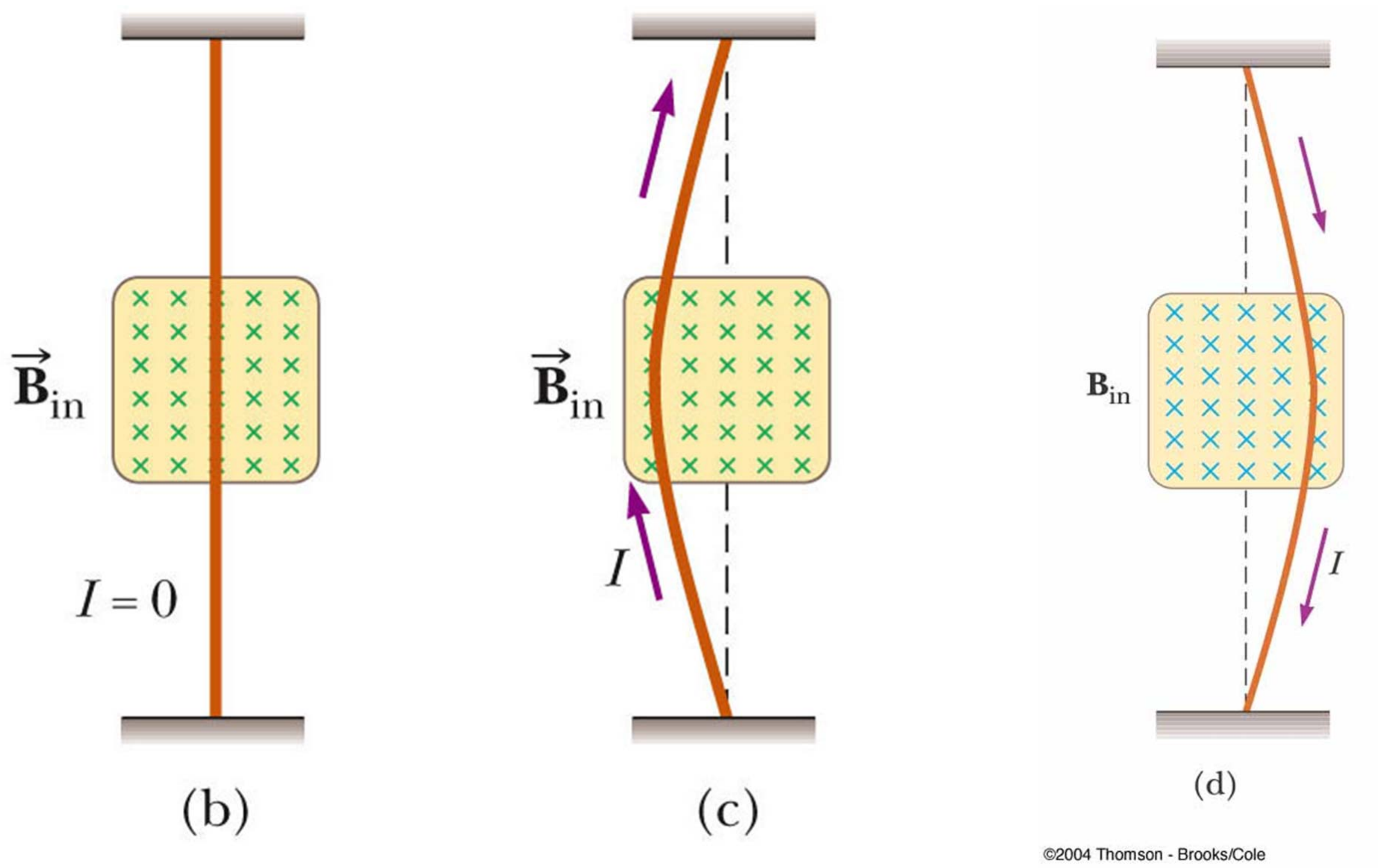


$$\frac{m}{q} = \frac{rB_0B}{E}$$

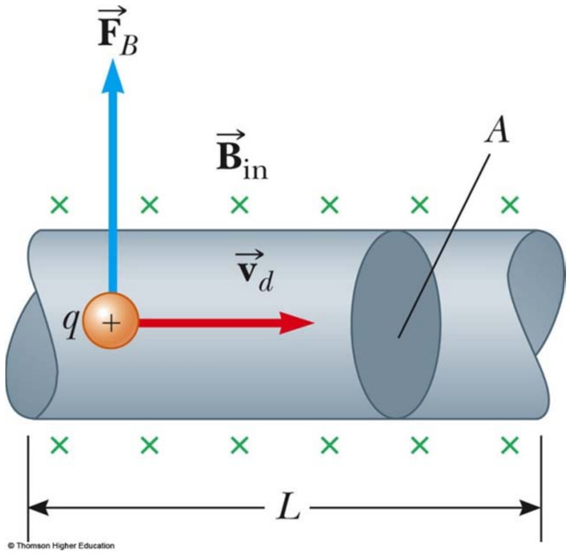
Magnetic Force on a Current Carrying Conductor

- Magnetic force acts upon charges moving in a conductor.
- The total force on the current is the integral sum of the force on each charge in the current.
- In turn, the charges transfer the force on to the wire when they collide with the atoms of the wire.

Magnetic Force on a Current Carrying Conductor



Magnetic Force on a Current Carrying Conductor



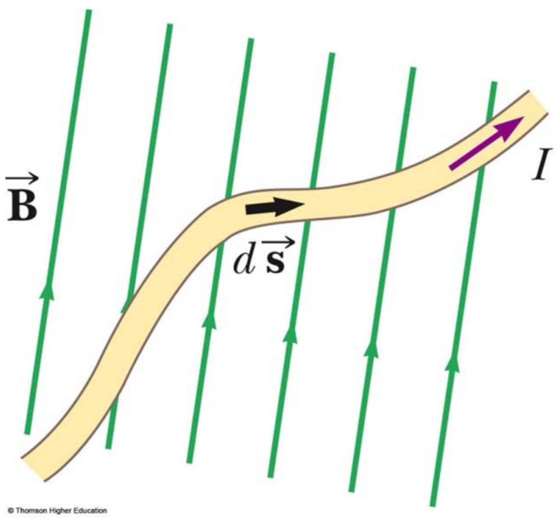
$$\vec{F}_{B,q} = \left(q\vec{v}_d \times \vec{B}\right)$$

$$\vec{F}_B = \left(q\vec{v}_d \times \vec{B}\right) nAL$$

$$I = v_d q n A$$

$$\vec{F}_B = I\vec{L} \times \vec{B}$$

For a straight wire in a uniform field

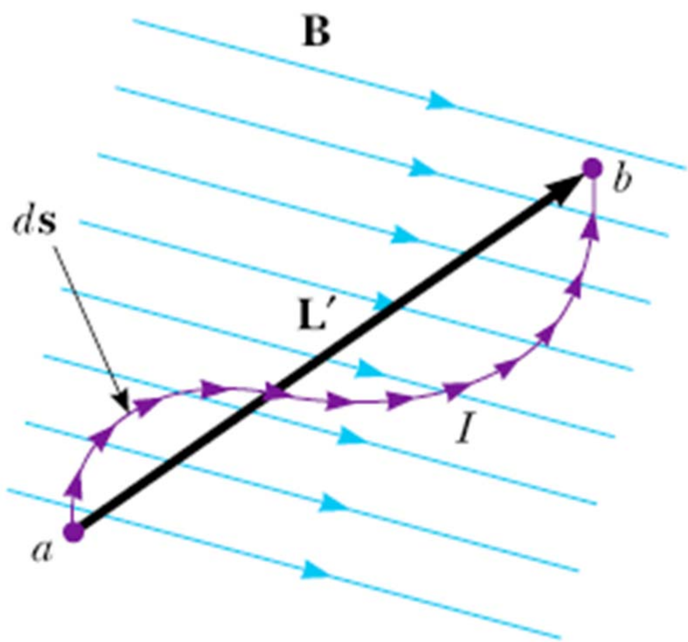


$$d\vec{F}_B = I d\vec{s} \times \vec{B}$$

$$\vec{F}_B = I \int_a^b d\vec{s} \times \vec{B}$$

For an arbitrary wire in an arbitrary field

A Special Case –
arbitrary wire in a uniform field



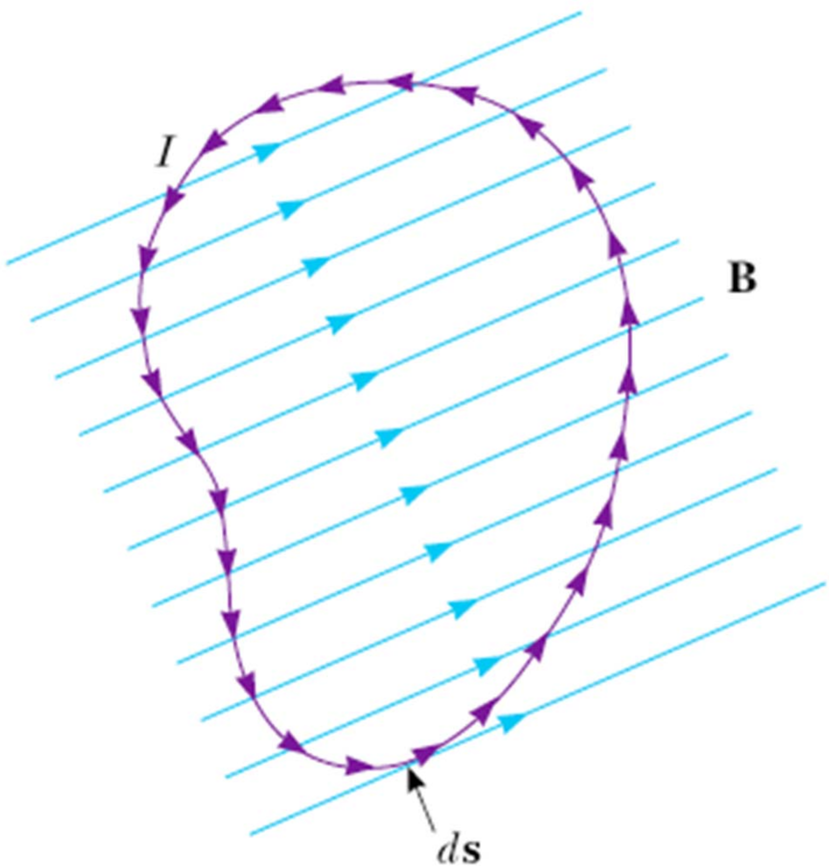
$$\vec{\mathbf{F}}_B = I\int_a^b d\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

$$\vec{\mathbf{F}}_B = I\left(\int_a^b d\vec{\mathbf{s}}\right) \times \vec{\mathbf{B}}$$

$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}}' \times \vec{\mathbf{B}}$$

The net magnetic force acting on a curved wire in a uniform magnetic field is the same as that of a straight wire between the same end points.

Closed Loop in a Magnetic Field



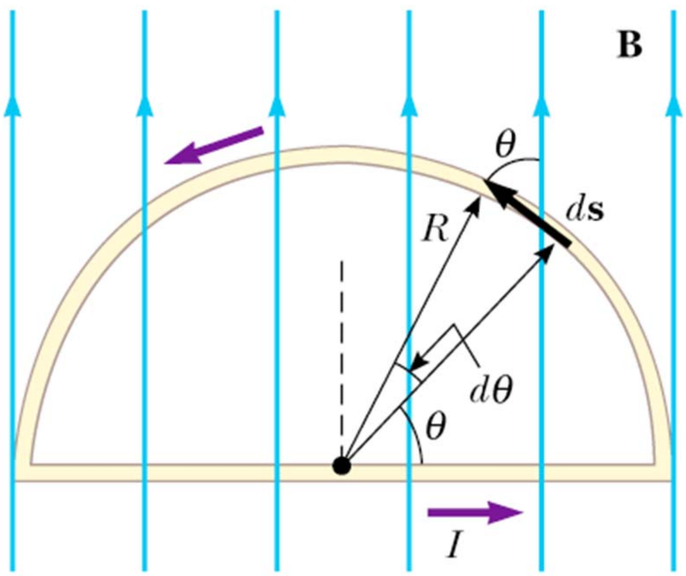
$$\vec{\mathbf{F}}_B = I\left(\oint d\vec{\mathbf{s}}\right)\times\vec{\mathbf{B}}$$

$$\oint d\vec{\mathbf{s}} = 0$$

$$\vec{\mathbf{F}}_B = 0$$

Net magnetic force acting on any closed current loop in a uniform magnetic field is zero.

Forces on a Semicircular Conductor



On the straight wire

$$F_1 = ILB = 2IRB$$

Directed out of the board

On the curved wire

$$dF_2 = I\left|d\vec{\mathbf{s}} \times \vec{\mathbf{B}}\right| = IB\sin\theta ds$$

$$s = R\theta$$

$$ds = Rd\theta$$

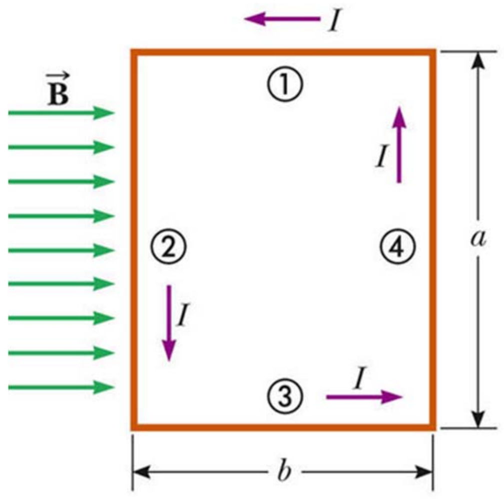
$$dF_2 = IRB\sin\theta d\theta$$

$$F_2 = \int_0^{\pi} IRB\sin\theta d\theta = IRB\int_0^{\pi} \sin\theta d\theta$$

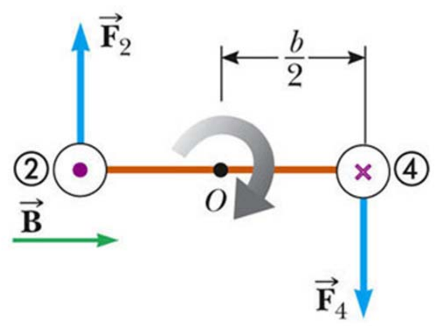
$$F_2 = 2IRB$$ Directed in to the board

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = 0$$

Torque on a Current Loop in a Uniform Magnetic Field



(a)



(b)

If the field is parallel to the plane of the loop

For 1 and 3, $\vec{\mathbf{L}} \times \vec{\mathbf{B}} = 0$

$\vec{\mathbf{F}}_B = 0$

For 2 and 4, $F_2 = F_4 = IaB$

$$\tau_{max} = F_2 \frac{b}{2} + F_4 \frac{b}{2} = (IaB)\frac{b}{2} + (IaB)\frac{b}{2}$$

$$\tau_{max} = (IaB)b$$

$$\tau_{max} = IAB$$

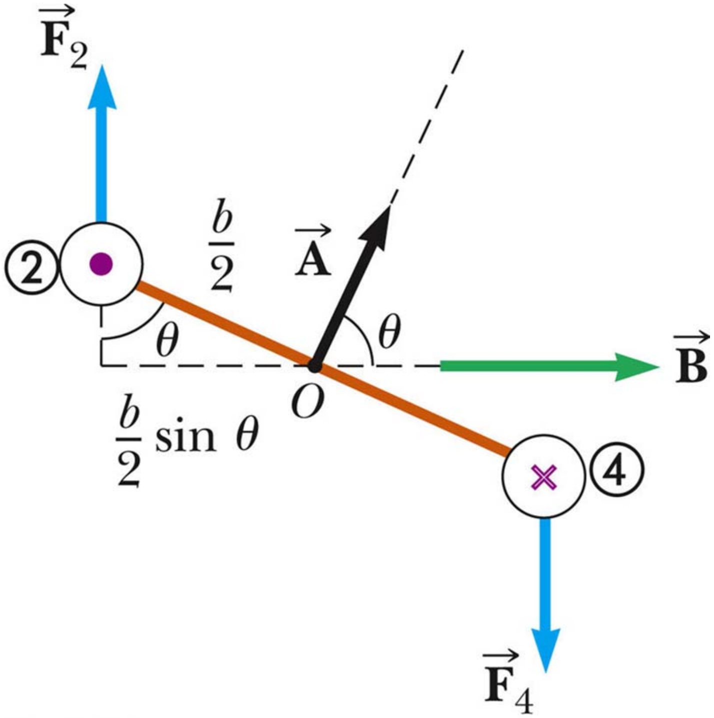
Torque on a Current Loop in a Uniform Magnetic Field

If the field makes an angle with a line perpendicular to the plane of the loop:

$$\tau = F_2 \frac{a}{2} \sin\theta + F_4 \frac{a}{2} \sin\theta$$

$$\tau = IaB\frac{b}{2} \sin\theta + IaB\frac{b}{2} \sin\theta = IabB\sin\theta$$

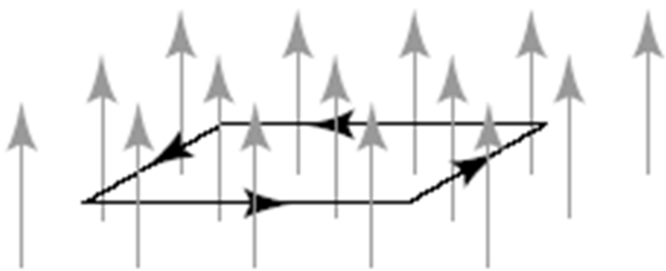
$$\tau = IAB\sin\theta$$



$$\vec{\boldsymbol{\tau}} = I\vec{\mathbf{A}} \times \vec{\mathbf{B}}$$

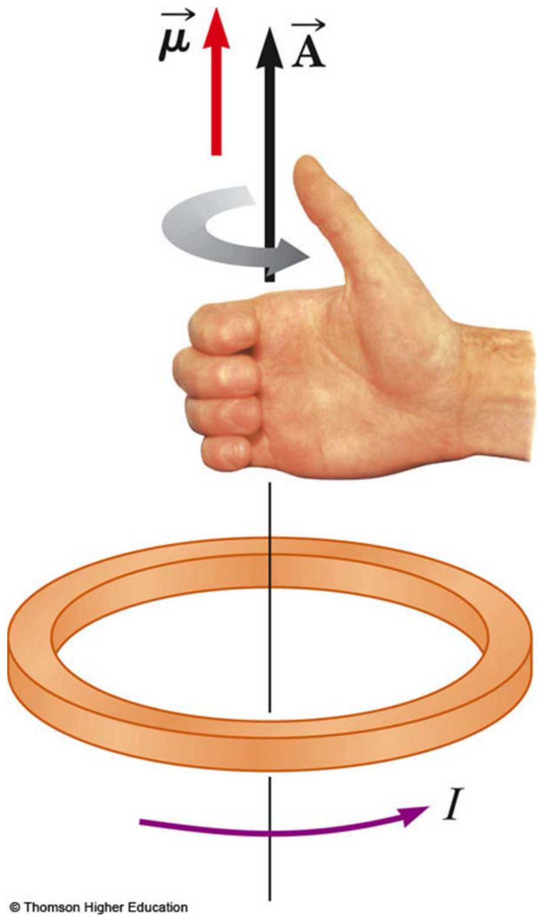
Concept Question

A rectangular loop is placed in a uniform magnetic field with the plane of the loop perpendicular to the direction of the field.
If a current is made to flow through the loop in the sense shown by the arrows, the field exerts on the loop:



1. a net force.
2. a net torque.
3. a net force and a net torque.
4. neither a net force nor a net torque.

Magnetic Dipole Moment



$$\vec{\mu} = I\vec{A}$$ (Amperes.m^2)

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

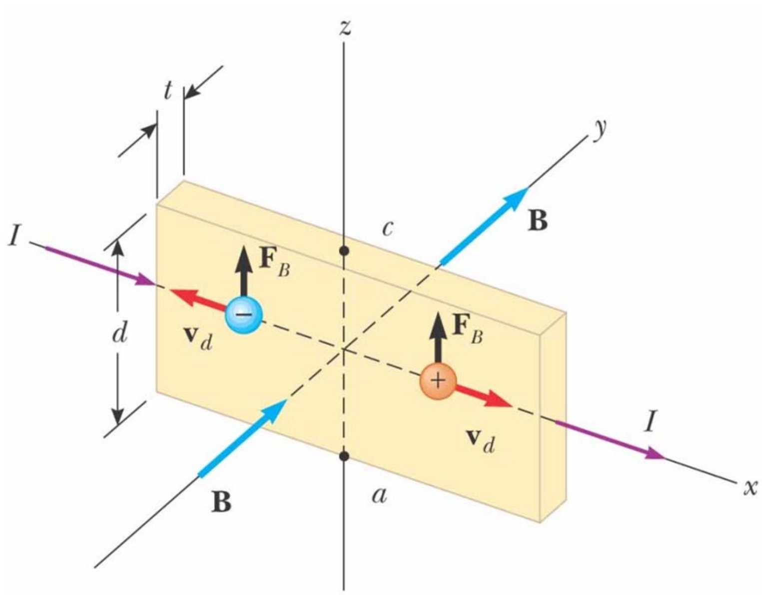
If the wire makes N loops around A,

$$\vec{\tau} = N\vec{\mu}_{loop} \times \vec{B} = \vec{\mu}_{coil} \times \vec{B}$$

The potential energy of the loop is:

$$U = -\vec{\mu} \cdot \vec{B}$$

Hall Effect



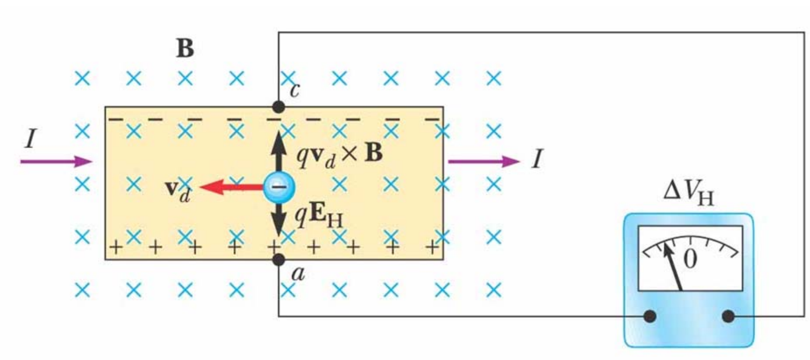
$$qv_d B = qE_H$$

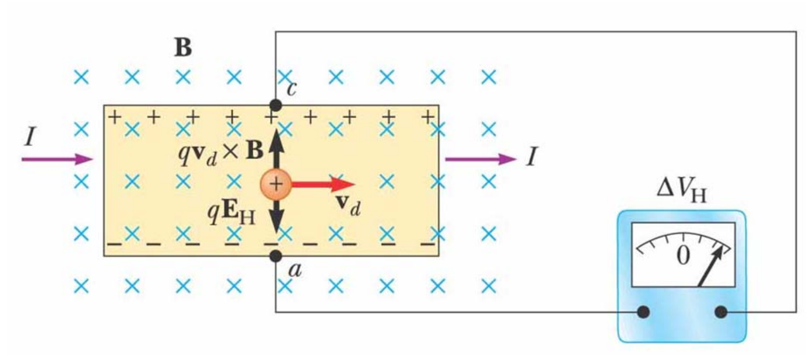
$$E_H = v_d B$$

$$\Delta V_H = v_d B d$$

$$v_d = \frac{I}{nqA}$$

$$\Delta V_H = \frac{IB}{nqt} = \frac{R_H IB}{t}$$





Summary

- The right hand rule
- Magnetic forces on charged particles
- Charged particle motion
- Magnetic forces on current carrying wires
- Torque on loops and magnetic dipole moments

For Next Class

- Reading Assignment
  - Chapter 30 – Sources of the Magnetic Field
- WebAssign: Assignment 7